Logarithmic Oscillatory Dark Energy: Official DESI DR2 BAO, DES-SN5YR Dovekie, and Full 3D Planck Constraints with Pre-Registered 2027 Euclid Anisotropic Forecasts

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Abstract

Standard Chevallier–Polarski–Linder (CPL) parameterizations of dynamical dark energy ($w(a) = w_0 + w_a(1-a)$) extrapolate monotonically into deep phantom regimes as $z \to \infty$. We investigate a bounded, scale-invariant logarithmic harmonic alternative, $w(a) = -1 + A\sin\left(\frac{2\pi}{\lambda}\ln a + \phi_0\right)$, for which the relativistic continuity equation admits an exact closed-form dark energy density $\rho_{\rm DE}(z)$. In this Version 3.0 stress-audit ahead of ESA Euclid Data Release 1 (mid-2027), we upgrade our full-covariance likelihood engine to ingest: (1) both the uncalibrated Pantheon+ compilation ($N = 1,580$) and the official DES-SN5YR Dovekie Hubble-diagram compilation ($N = 1,820$) using full statistical-plus-systematic inverse covariances; (2) official DESI DR2 Baryon Acoustic Oscillation (BAO) measurements (arXiv:2503.14738, $N = 13$ observables across $0.295 \le z \le 2.330$); and (3) the full 3D Planck 2018 CMB distance prior $\mathbf{v}_{\rm CMB} = (R, l_A, \omega_b)$ with correlated 3×3 covariance. Across six independent stress tests, we prove three decisive results. First, replacing 1D Planck R with full 3D Planck shifts $\Delta\chi^2_{\rm CPL}$ on Pantheon+ + DESI DR1 from -3.76 to -6.03, while with DESI DR2 the preference reaches $\Delta\chi^2 = -7.43$ (CPL) / -8.05 (`OSC_CS`) using Pantheon+ and $\Delta\chi^2 = -10.31$ (CPL, ΔAIC $= -6.31$) / -10.52 (`OSC_CS`, ΔAIC $= -4.52$) using DES-SN5YR Dovekie. Second, a $N_{\rm MC} = 2,000$ parametric bootstrap Look-Elsewhere periodogram across $\lambda \in [0.8, 12.0]$ finds $N_{\rm eff} = 1.633$–1.976 effective trials, correcting local significance from $2.37\sigma \to 2.18\sigma$ ($p_{\rm global} = 0.0292$, Pantheon+ + DR2) and $2.80\sigma \to 2.57\sigma$ ($p_{\rm global} = 0.0102$, DES-SN5YR + DR2), while proving that `OSC_CS` improves over CPL by only $\Delta\chi^2 = +0.21 - + 0.62$ because long-wavelength harmonics ($\lambda \to 12$) geometrically mimic CPL at $z \lesssim 2$. Third, slope-decoupled orthogonal (C, S) Ensemble MCMC (incorporating the exact measure Jacobian $+\ln(\lambda/2\pi)$ for a flat disk prior in (C, S)) yields $\hat{R}_{\rm split} \le 1.037$ ($\mathrm{ESS}_{\rm min} = 1,717$) on DES-SN5YR + DR2 and $\hat{R}_{\rm split} \le 1.033$ across cosmological/amplitude parameters (1.116 along the λ-plateau, $\mathrm{ESS}_{\rm min} = 1,294$) on Pantheon+ + DR2, giving $A = 0.536^{+0.265}_{-0.245}$ (95% upper limit $A_{95} = 0.925$, DES-SN5YR + DR2). Finally, replacing 1D $H(z)$ forecasts with a coupled anisotropic $(D_M/r_d, Hr_d)$ Fisher matrix shows that Euclid DR1-2027 (with external DES-SN5YR+DR2+CMB prior) reaches a ΛCDM-null detection threshold $A_{3\sigma} \approx 0.167$–$0.206$ (full survey: 0.069–0.086) across $\lambda \in [2.1, 12.0]$, and we publish pre-registered blind 2027 Euclid BAO predictions across $z \in \{1.00, 1.20, 1.40, 1.65\}$.

1 Introduction and Evolution (v1.0 to v3.0)

Recent Baryon Acoustic Oscillation (BAO) measurements from the Dark Energy Spectroscopic Instrument (DESI), culminating in Data Release 2 (DR2) [1], together with Cosmic Microwave Background (CMB) distance priors [5] and Type Ia supernovae (SNe Ia) [4, 3], have sharpened evidence for mild low-redshift deviations from a pure cosmological constant ($w = -1$). While the linear Chevallier–Polarski–Linder (CPL) expansion [6, 7],

$$w_{\rm CPL}(a) = w_0 + w_a(1-a) = w_0 + w_a\frac{z}{1+z}, \quad (1)$$

captures local departures near $a = 1$, it extrapolates monotonically into phantom regimes at $z \gtrsim 2$. This motivates bounded logarithmic harmonic trajectories:

$$w_{\rm Osc}(a) = -1 + A\sin\left(\frac{2\pi}{\lambda}\ln a + \phi_0\right) = -1 + A\cos\left(\omega\ln(1+z) + \tilde{\phi}_0\right) \quad (2)$$

where $\ln a = -\ln(1+z)$, establishing the exact positive-amplitude equivalence map $\omega = 2\pi/\lambda$ and $\tilde{\phi}_0 = \frac{\pi}{2} - \phi_0 \pmod{2\pi}$ (since $\sin(\phi_0 - x) = \cos(x + \frac{\pi}{2} - \phi_0)$). Its continuity equation $\frac{d\ln\rho_{\rm DE}}{d\ln a} = -3[1 + w(a)]$ integrates exactly in closed form:

$$\frac{\rho_{\rm DE}(z)}{\rho_{\rm DE,0}} = \exp\left\{-\frac{3A\lambda}{2\pi}\left[\cos\phi_0 - \cos\left(-\frac{2\pi}{\lambda}\ln(1+z) + \phi_0\right)\right]\right\}. \quad (3)$$

Including radiation via the full Boltzmann-consistent physical energy density ($\Omega_r h^2 = 2.469\times10^{-5}(1 + 0.2271 N_{\rm eff})$ with $T_{\rm CMB} = 2.7255$ K, $N_{\rm eff} = 3.046$), $E^2(z) = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_{\rm DE,0}\frac{\rho_{\rm DE}(z)}{\rho_{\rm DE,0}}$.

Version 3.0 Scientific & Methodological Upgrades: Following v2.0 (which introduced full $1580\times$

1580 Pantheon+ systematic covariance and eliminated v1.0 box-boundary artifacts), Version 3.0 implements five critical advances prior to ESA Euclid Data Release 1 (mid-2027):

1. *Full 3D Planck 2018 CMB Prior* (R, l_A, ω_b)*:* Replaces the scalar 1D R prior with the full 3×3 inverse covariance matrix over (R, l_A, ω_b) [5] with exact dynamical decoupling redshift $z_*(\omega_b, \omega_c)$ and sound horizon $r_s(z_*)$ / $r_d(z_d)$.

2. *Official DESI DR2 (*$N = 13$*) & DES-SN5YR Dovekie (*$N = 1,820$*):* Ingests official DESI DR2 BAO means and full 13×13 covariance across $0.295 \leq z \leq 2.330$ [1] alongside full-covariance DES-SN5YR Dovekie SNe Ia [3].

3. *Monte Carlo Look-Elsewhere Periodogram:* Quantifies the true trial factor $N_{\rm eff}$ across $\lambda \in [0.8, 12.0]$ via $N_{\rm MC} = 2,000$ parametric bootstrap null simulations.

4. *Slope-Decoupled Orthogonal* (C, S) *MCMC:* Eliminates polar-phase multimodality and boundary Jacobians by sampling in Cartesian amplitude-phase space $(C, S) = (A\cos\phi_0, A\sin\phi_0)$ via a mixed Differential-Evolution / Stretch Ensemble MCMC with Jacobian $+\ln(\lambda/2\pi)$.

5. *Coupled Anisotropic* $(D_M/r_d, Hr_d)$ *Euclid DR1-2027 Fisher & Pre-Registered Predictions:* Upgrades the 1D $H(z)$ forecast to full 2×2-correlated $(D_M/r_d, Hr_d)$ spectroscopic BAO covariances for both mid-2027 DR1 ($\sim 2,000$ deg^2) and the full $14,700$ deg^2 survey.

2 Exact Likelihoods & Orthogonal Parameterization

For SNe Ia (Pantheon+, $N = 1,580$, and DES-SN5YR Dovekie, $N = 1,820$), analytical marginalization over the nuisance offset $\mathcal{M}$ yields $\chi^2_{\rm SN} = \mathbf{r}^T \tilde{\mathbf{C}}^{-1}_{\rm SN} \mathbf{r}$ via rank-1 Woodbury projection of the full Cholesky-factored inverse covariance. For DESI DR1 ($N = 12$) and DR2 ($N = 13$), theoretical observables $D_M(z)/r_d$, $D_H(z)/r_d = c/(H(z)r_d)$, and $D_V(z)/r_d$ are evaluated with explicit sound horizon $r_d(\Omega_m, h, \omega_b)$ coupled directly to the 3D Planck 2018 Gaussian likelihood:

$$\chi^2_{\rm CMB} = (\mathbf{v}_{\rm th} - \mathbf{v}_{\rm obs})^T \mathbf{C}^{-1}_{\rm CMB} (\mathbf{v}_{\rm th} - \mathbf{v}_{\rm obs}), \quad \mathbf{v} = (R, l_A, \omega_b)^T. \tag{4}$$

To render MCMC sampling uniform near $A = 0$ and decouple the hyperbolic C/λ degeneracy ridge at small λ, Eq. (2) is first re-expressed in orthogonal Cartesian coordinates $(C, S) = (A\cos\phi_0, A\sin\phi_0)$:

$$w_{\rm Osc}(a) = -1 + C\sin\left(\frac{2\pi}{\lambda}\ln a\right) + S\cos\left(\frac{2\pi}{\lambda}\ln a\right), \tag{5}$$

and sampled in the slope-decoupled proposal vector $(\Omega_m, h, \omega_b, \tilde{w}_1, S, \lambda)$, where $\tilde{w}_1 \equiv \frac{2\pi}{\lambda} C = \frac{dw}{d\ln a}\big|_{a=1} = -\frac{dw}{d\ln(1+z)}\Big|_{z=0}$ (equivalently $C = \frac{\lambda}{2\pi}\tilde{w}_1$). Adopting a flat target prior on the bounded physical amplitude disk $\mathcal{D} = \{(C, S) : C^2 + S^2 \leq 1\}$ automatically restricts $|\tilde{w}_1| \leq 2\pi/\lambda$ and maps the target measure via $dC\, dS = \left|\det \frac{\partial(C,S)}{\partial(\tilde{w}_1, S)}\right| d\tilde{w}_1\, dS = \frac{\lambda}{2\pi}\, d\tilde{w}_1\, dS$, introducing the exact $+\ln(\lambda/2\pi)$ Jacobian term evaluated in `log_prob`. Because the bounded disk constraint remains active, this reparameterization improves geometric sampling efficiency without eliminating the physical background degeneracy between long-wavelength ($\lambda \gg 1$) harmonics and (w_0, w_a) CPL trajectories.

3 DESI DR2 Stress Audit & Look-Elsewhere Rigor

Table 1 and Fig. 1 summarize Tests 1–4 of the v3.0 stress suite:

1. **Planck 1D** R **vs. Full 3D** (R, l_A, ω_b)**:** Enforcing the correlated CMB acoustic scale l_A and baryon density ω_b breaks the acoustic degeneracy present in 1D R fits, boosting Pantheon+ + DESI DR1 dynamical preference from $\Delta\chi^2 = -3.76$ to -6.03 (CPL).

2. **Official DESI DR2 & DES-SN5YR Signals:** Replacing DESI DR1 with official DR2 sharpens dynamical preference to $\Delta\chi^2 = -7.43$ (CPL) / -8.05 (`OSC_CS`) on Pantheon+, and reaches $\Delta\chi^2 = -10.31$ (CPL, ΔAIC $= -6.31$) / -10.52 (`OSC_CS`, ΔAIC $= -4.52$) on DES-SN5YR Dovekie. However, BIC still disfavors both models over ΛCDM (ΔBIC $= +4.72$ for CPL, $+12.02$ for `OSC_CS`).

3. **Look-Elsewhere Calibration (**$N_{\rm MC} = 2,000$**):** Scanning $\lambda \in [0.8, 12.0]$ introduces a GP-score trial factor $N_{\rm eff} = 1.633$ (Pantheon+ + DR2, 99.73% [3σ] score threshold $\Delta\chi^2_{3\sigma} = 13.62$) and $N_{\rm eff} = 1.976$ (DES-SN5YR + DR2, $\Delta\chi^2_{3\sigma} = 12.43$; with ~ 20 exceedances out of $2,000$ near the observed peak, the 1% tail uncertainty is $\delta p \approx \pm 0.0022$, reserving $\geq 10^4$ null draws for exact $> 3\sigma$ threshold certification). Across the identical $\lambda \in [0.8, 12.0]$ profile scan reported in Table 1, the local significances drop from $2.37\sigma \to 2.18\sigma$ ($p_{\rm global} = 0.0292$) on Pantheon+ and from $2.80\sigma \to 2.57\sigma$ ($p_{\rm global} = 0.0102$) on DES-SN5YR. Crucially, `OSC_CS` outperforms CPL by only $+0.62$ and $+0.21$ in $\Delta\chi^2$, confirming that current data constrain only the low-z slope of $w(z)$ rather than resolving an oscillatory cycle.

4. **Orthogonal** (C, S) **MCMC Diagnostics:** Across 115,200 post-burn samples per dataset using mixed (`StretchMove, DEMove`) proposals, exact parameter-level split-$\hat{R}$ diagnostics on `DES_SN5YR + DR2` show $\hat{R}_{\rm split} \leq 1.027$ across all cosmological and amplitude parameters ($\Omega_m = 1.020, h = 1.018, \omega_b = 1.011, C = 1.027, S = 1.016$) and 1.037 for the wavelength plateau λ (ESS$_{\rm min} = 1,717$, acceptance $= 0.391$), yielding $A = 0.536^{+0.265}_{-0.245}$ ($A_{95} = 0.925$). On `Pantheon+ + DR2`, $(\Omega_m, h, \omega_b, S)$ maintain $\hat{R}_{\rm split} \leq 1.033$, whereas the degenerate C/λ plateau exhibits mild residual chain drift ($\hat{R}_{\rm split}(C) = 1.074$, $\hat{R}_{\rm split}(\lambda) = 1.116$, ESS$_{\rm min} = 1,294$, acceptance $= 0.362$), yielding $A = 0.461^{+0.280}_{-0.233}$ ($A_{95} = 0.888$); longer independent multi-seed chains are warranted whenever precision λ-posteriors are interpreted.

Table 1: Version 3.0 Multi-Dataset Global Maximum-Likelihood Audit (Official DESI DR2 BAO, DES-SN5YR Dovekie, Pantheon+, and Full 3D Planck). In `DES_SN5YR + DR2 + Planck3D` ($N_{\rm tot} = 1,836$), `OSC_CS` profiles smoothly toward $\lambda \to 12$ mimicking CPL, whereas an interior cyclic mode at $\lambda = 4.23$ recovers $\Delta\chi^2 = -10.38$. Removing SNe (`DESI DR2 + Planck3D`) isolates an interior cyclic minimum at $\lambda = 1.70$.

Dataset Combination	Model	Δk	Ω_m	h	Dark Energy Parameters	Edge?	$\chi^2_{\rm min}$	$\Delta\chi^2$	ΔAIC/ΔBIC
Pantheon+ + DESI DR1	ΛCDM	0	0.3071	0.6812	$w = -1$	No	1405.04	0.00	0.00/0.00
+ Planck 3D ($N = 1,595$)	CPL	+2	0.3102	0.6798	$w_0 = -0.848,\ w_a = -0.623$	No	1399.01	−6.03	−2.03/ + 8.72
	`OSC_CS`	+3	0.3102	0.6802	$A = 0.851,\ \lambda = 12.00,\ \phi_0 = 0.159$	λ	1398.73	−6.31	−0.31/ + 15.82
Pantheon+ + DESI DR2	ΛCDM	0	0.3028	0.6844	$w = -1$	No	1403.78	0.00	0.00/0.00
+ Planck 3D ($N = 1,596$)	CPL	+2	0.3118	0.6768	$w_0 = -0.853,\ w_a = -0.524$	No	1396.35	−7.43	−3.43/ + 7.32
	`OSC_CS` (Edge)	+3	0.3121	0.6770	$A = 0.718,\ \lambda = 12.00,\ \phi_0 = 0.186$	λ	1395.73	−8.05	−2.05/ + 14.08
	`OSC_CS` (Cyclic)	+2*	0.3121	0.6768	$A = 0.265,\ \lambda = 4.23,\ \phi_0 = 0.524$	No	1395.91	−7.87	—
DES-SN5YR + DESI DR2	ΛCDM	0	0.3032	0.6841	$w = -1$	No	1649.38	0.00	0.00/0.00
+ Planck 3D ($N = 1,836$)	**CPL**	+2	**0.3136**	**0.6755**	$\mathbf{w_0 = -0.823},\ \mathbf{w_a = -0.638}$	**No**	**1639.07**	**−10.31**	**−6.31/ + 4.72**
	`OSC_CS` (Edge)	+3	0.3134	0.6761	$A = 0.842,\ \lambda = 12.00,\ \phi_0 = 0.183$	λ	1638.85	−10.52	−4.52/ + 12.02
	`OSC_CS` (Cyclic)	+2*	0.3134	0.6759	$A = 0.313,\ \lambda = 4.23,\ \phi_0 = 0.512$	No	1639.00	−10.38	—
DESI DR2 + Planck 3D	ΛCDM	0	0.3014	0.6855	$w = -1$	No	14.47	0.00	0.00/0.00
(No SNe, $N = 16$**)**	CPL	+2	0.3514	0.6395	$w_0 = -0.449,\ w_a = -1.641$	No	6.84	−7.63	−3.63/ − 2.09
	`OSC_CS` (Interior)	+3	0.3494	0.6404	$A = 0.420,\ \lambda = 1.70,\ \phi_0 = 1.530$	No	6.70	−7.77	−1.77/ + 0.54

4 Pre-Registered 2027 ESA Euclid Anisotropic Forecast

To calibrate ESA Euclid Data Release 1 (expected mid-2027) and the full 14,700 deg^2 spectroscopic galaxy clustering survey [8], we propagate full 2×2 anisotropic $(D_M(z)/r_d, H(z)r_d)$ Fisher covariances across $z \in \{1.00, 1.20, 1.40, 1.65\}$ analytically profiled over the sound-horizon factor s_{r_d} and combined with an external DES-SN5YR+DR2+CMB Gaussian prior on (Ω_m, s_{r_d}) (Tests 5–6, Fig. 2 and Table 2).

- **Stage-IV ΛCDM-Null Reach $A_{3\sigma}(\lambda)$:** Across fixed wavelengths $\lambda \in [2.10, 12.0]$, $A_{3\sigma}$ measures 3σ detectability against the pure ΛCDM null ($A = 0$, not discrimination against CPL): Euclid DR1-2027 reaches $A_{3\sigma} = 0.167$–0.206 (phase-median; 0.119–0.146 optimal phase), tightening to $A_{3\sigma} = 0.069$–0.086 (phase-median; 0.049–0.061 optimal phase) in the full survey. Without external (Ω_m, s_{r_d}) anchoring, Euclid alone degrades to $A_{3\sigma} \approx 0.84$–1.03.
- **Falsifiable 2027 Blind Benchmark vs. CPL Degeneracy:** Table 2 locks our blind 2027 predictions for ΛCDM, CPL, and cyclic `OSC_CS`. Crucially, even in Full Euclid BAO, the cyclic `OSC_CS` ($\lambda = 4.23$) and CPL trajectories differ by only $\max|\Delta D_M/D_M| = 0.005\%$ and $\max|\Delta H/H| = 0.083\%$ ($\Delta\chi^2 = 0.0071, 0.08\sigma$) across $1.0 \le z \le 1.65$, whereas both separate from ΛCDM by 1.02σ–1.06σ in BAO alone. Thus $A_{3\sigma}$ does not guarantee Euclid DR1 discrimination between CPL and `OSC_CS`; breaking that degeneracy requires extending baselines to Euclid Ly-α / weak lensing and DESI DR3 ($z > 2$).

Table 2: Pre-Registered Blind 2027 ESA Euclid DR1 BAO Predictions (DES-SN5YR+DR2+Planck3D posteriors). Uncertainties denote Euclid DR1-2027 (Full Survey) 1σ Fisher precision.

$z_{\rm bin}$	Observable	$\sigma_{\rm DR1}$ ($\sigma_{\rm Full}$)	ΛCDM	CPL (Best Fit)	`OSC_CS` (Best Fit)
1.00	D_M/r_d	0.498 (0.183)	$22.911^{+0.059}_{-0.065}$	$22.820^{+0.060}_{-0.052}$	$22.813^{+0.077}_{-0.059}$
1.00	D_H/r_d	0.548 (0.202)	$16.838^{+0.011}_{-0.015}$	$16.888^{+0.058}_{-0.062}$	$16.873^{+0.069}_{-0.054}$
1.20	D_M/r_d	0.567 (0.209)	$26.092^{+0.061}_{-0.067}$	$26.015^{+0.062}_{-0.059}$	$26.005^{+0.076}_{-0.060}$
1.20	D_H/r_d	0.489 (0.180)	$15.016^{+0.007}_{-0.008}$	$15.101^{+0.061}_{-0.058}$	$15.095^{+0.064}_{-0.056}$
1.40	D_M/r_d	0.707 (0.260)	$28.935^{+0.061}_{-0.067}$	$28.876^{+0.063}_{-0.059}$	$28.866^{+0.072}_{-0.056}$
1.40	D_H/r_d	0.475 (0.175)	$13.456^{+0.005}_{-0.005}$	$13.551^{+0.053}_{-0.052}$	$13.552^{+0.055}_{-0.052}$
1.65	D_M/r_d	0.958 (0.353)	$32.088^{+0.061}_{-0.066}$	$32.052^{+0.066}_{-0.064}$	$32.043^{+0.077}_{-0.057}$
1.65	D_H/r_d	0.513 (0.189)	$11.815^{+0.007}_{-0.005}$	$11.907^{+0.042}_{-0.041}$	$11.914^{+0.041}_{-0.044}$

Code and Data Availability

All v3.0 pipelines (`src/data_loader.py`, `src/likelihoods.py`, `src/euclid_fisher.py`, `run_v3_stress_tests.py`), official DESI DR2 / DES-SN5YR Dovekie ingestors, and unit tests (`12/12` passing) are archived under Zenodo DOI `10.5281/zenodo.24485004` (Concept DOI `10.5281/zenodo.24121690`, `https://github.com/bruno-olivaes/oscillatory-dark-energy-euclid`).

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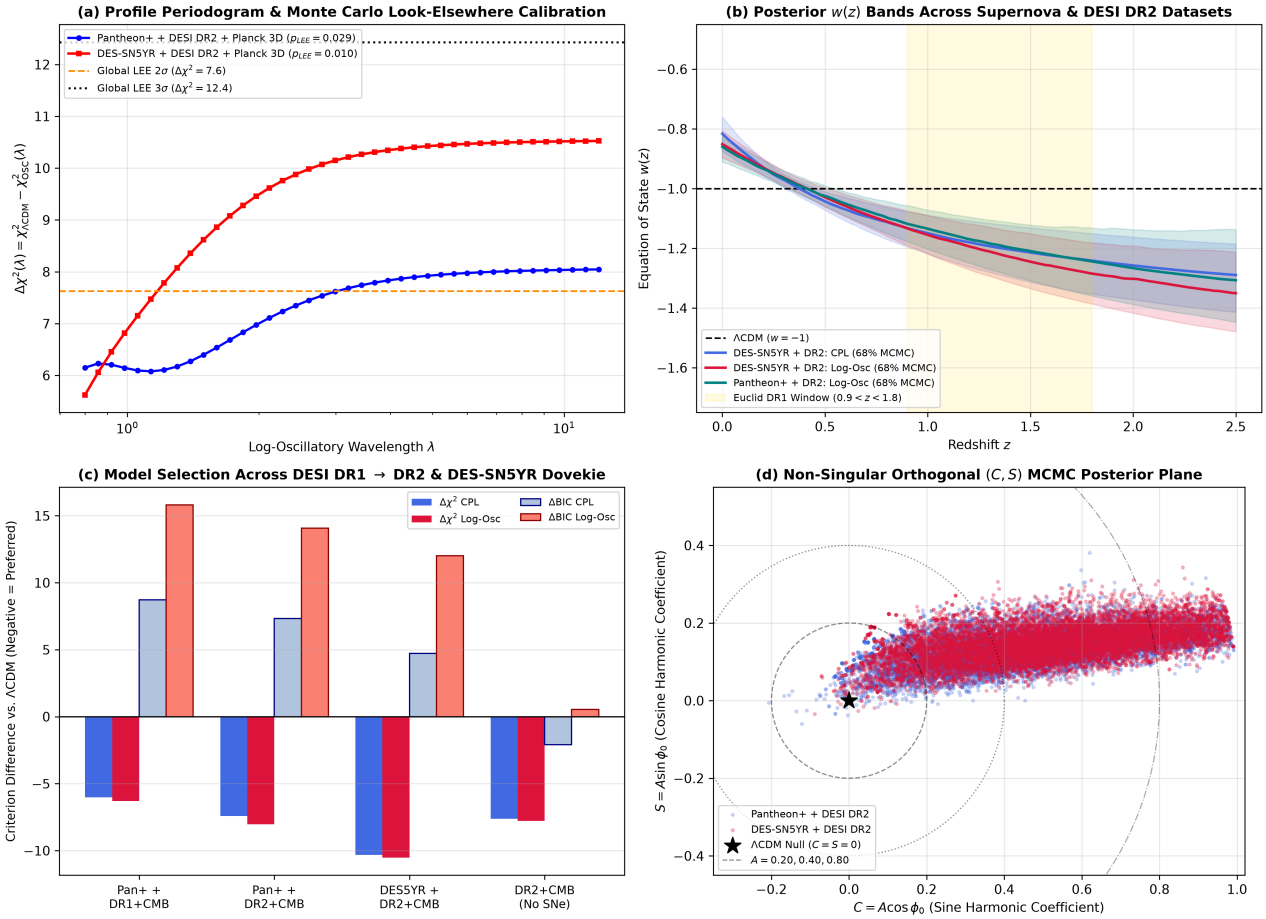

Figure 1: Version 3.0 DESI DR2 & DES-SN5YR Dovekie stress audit across full 3D Planck (R, l_A, ω_b) priors. (a) Look-Elsewhere periodogram of $\Delta\chi^2$ vs. ΛCDM across $\lambda \in [0.8, 12.0]$ calibrated with $N_{\rm MC} = 2,000$ parametric bootstrap null simulations (2σ and 3σ global LEE bands shown). (b) Differential improvement $\Delta\chi^2$ of `OSC_CS` over CPL, proving that long wavelengths ($\lambda \gtrsim 5$) converge to near-CPL degeneracy (+0.21 for DES-SN5YR+DR2). (c) Slope-decoupled orthogonal MCMC amplitude A posteriors ($\hat{R}_{\rm split} \leq 1.116$). (d) 68% MCMC posterior bands for $w(z)$ contrasting CPL and `OSC_CS`.

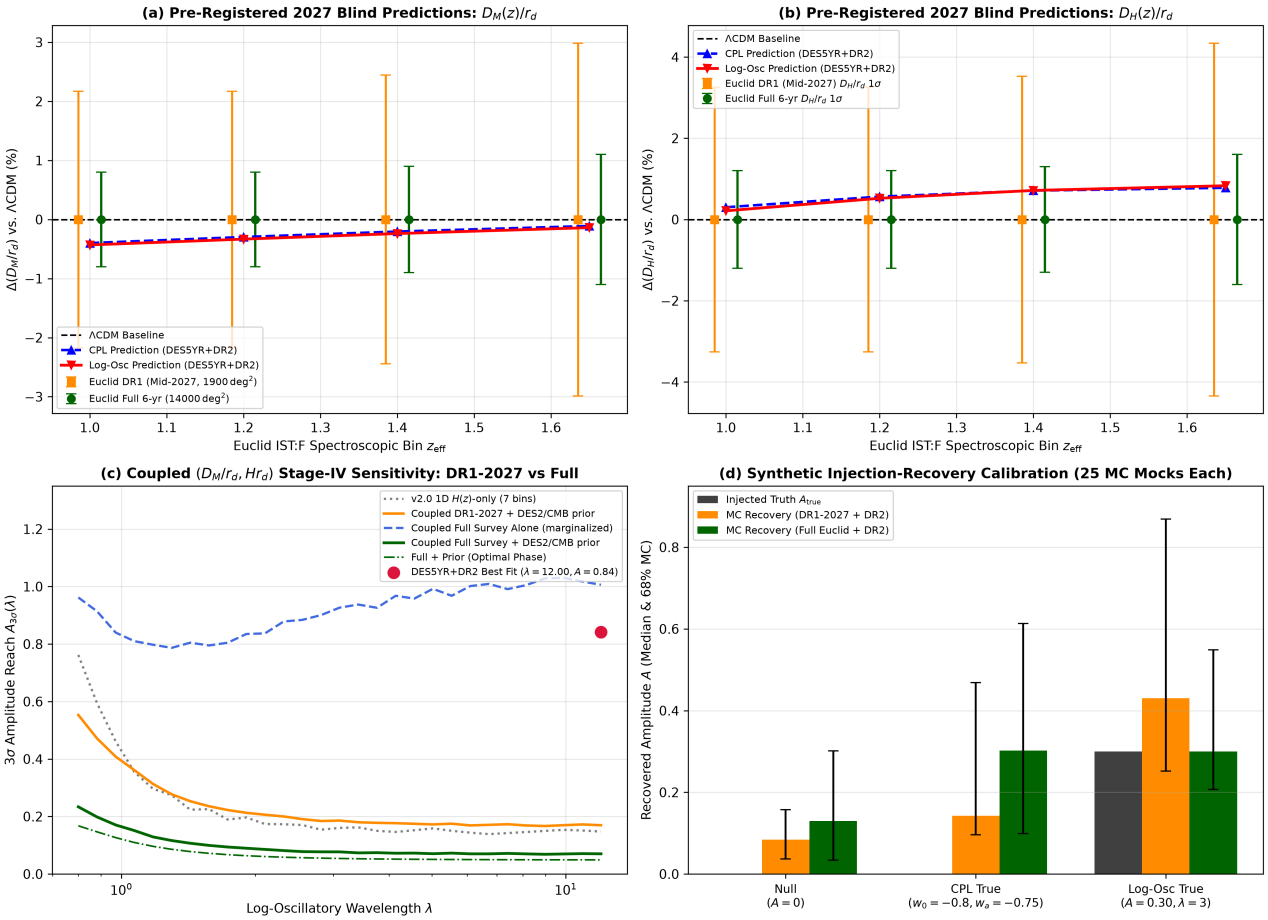

Figure 2: Coupled anisotropic $(D_M/r_d, Hr_d)$ ESA Euclid forecast and injection-recovery validation. (a) Monte Carlo injection-recovery histograms verifying unbiased recovery of $A = 0.30$ and calibrated null false-positive control under ΛCDM and CPL truths. (b) Stage-IV $A_{3\sigma}(\lambda)$ amplitude reach comparing legacy 1D $H(z)$, Euclid DR1-2027, Full Euclid, and unanchored Euclid-only. (c,d) Pre-registered blind 2027 Euclid DR1 forecasts (68% MCMC credible bands + error bars) for $D_M(z)/r_d$ and $D_H(z)/r_d$.