When Harmonic ZNE Fails: Equal-Budget, Out-of-Family Noise, and Optimizer-Instability Audits on NISQ Devices

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Abstract

Zero-Noise Extrapolation (ZNE) is widely used on Noisy Intermediate-Scale Quantum (NISQ) processors, yet head-to-head comparisons frequently omit equal-shot-budget unmitigated baselines, out-of-family noise stress tests, and non-linear optimizer instability rates. Here we evaluate polynomial, exponential, forced damped harmonic, stability-gated automatic Lomb–Scargle (`Auto_LombScargle_ZNE`), and two-qubit Clifford Pauli-Twirled ZNE across four 1-qubit noise/schedule regimes (A1–A4, $K = 25$ seeds, up to $N_{\text{total}} = 32{,}768$ shots/seed) and an entangled 4-qubit Transverse-Field Ising Model (TFIM) Trotter circuit (3 steps, 18 CX gates, $K = 20$ seeds, $N_{\text{total}} = 24{,}000$ shots/seed) in Qiskit 2.x (`AerSimulator`). In the in-family deep 1-qubit coherent Ramsey benchmark (A1, $s \leq 17$), both forced Harmonic ZNE and stability-gated `Auto_LombScargle_ZNE` reduce Mean Absolute Error (MAE) to 0.0084 ± 0.0012 vs. 0.0482 ± 0.0004 for the equal-shot unmitigated baseline (-82.6%, $z = +31.74\,\text{SE}$, 0% instability). Conversely, under out-of-family perturbations—short coherent scales (A2, $s \leq 6$), monotonic amplitude damping plus depolarizing relaxation (A3), and bi-chromatic quasi-static Gaussian drift (A4)—forced Harmonic ZNE exhibits 100% optimizer instability (MAE = 0.2007–0.3601). In 4-qubit TFIM circuits, forced Harmonic ZNE collapses with 100% instability at every coupling strength (MAE = 2.2868 at $\epsilon = 0.04$ rad vs. 0.1450 ± 0.0008 equal-budget unmitigated). Stability-gating (`unstable_fit` combining covariance condition number $> 10^4$ and 2% bound saturation) eliminates harmonic divergence (0% instability across A2–A4 and TFIM), while Pauli Twirling + ZNE reduces 4-qubit MAE by 75.2%–81.3% vs. the equal-budget unmitigated baseline—and maintains 86.8% reduction (0.0383 ± 0.0070 vs. 0.2911 ± 0.0012, $z = +35.55\,\text{SE}$) even when single-qubit twirl frame gates (`x, y, z`) carry realistic depolarizing and coherent errors.

1 Introduction

Current quantum hardware operates in the Noisy Intermediate-Scale Quantum (NISQ) regime [1], where gate errors reflect both stochastic decoherence and systematic coherent drift (residual Stark shifts, calibration detuning, and ZZ crosstalk) [5, 7]. Zero-Noise Extrapolation (ZNE) [2, 3, 4] estimates noiseless expectation values without ancilla qubits by running circuits across amplified noise scales $\lambda_j \geq 1$ and extrapolating to $\lambda \to 0$.

When coherent noise accumulates periodically with λ, low-order polynomials and pure exponentials can fail or invert sign. Spectral extensions based on the Lomb–Scargle periodogram [8, 9] attempt to fit damped harmonic trajectories $E(\lambda) = E_0 e^{-\gamma\lambda}\cos(\omega\lambda + \phi)$, whereas Clifford Pauli Twirling [6, 7] tailors coherent two-qubit errors into stochastic Pauli channels prior to ZNE.

Evaluating mitigation honestly requires three methodological controls often missing from proof-of-concept benchmarks: (1) an **equal-shot-budget unmitigated baseline** spending the entire shot budget N_{total} on $\lambda = 1$ alongside an equal-budget **twirl-only baseline** without extrapolation, plus explicit compiled-circuit count tracking (1 vs. 6 vs. 12 vs. 48 circuits/seed); (2) **out-of-family noise channels and restricted scale horizons** outside the mono-mode sinusoidal ansatz; and (3) explicit **nonlinear optimizer stability auditing** and finite Pauli-frame error robustness. Version 3.0 addresses all three requirements quantitatively.

2 Methodology and Mitigation Architectures

2.1 Transmon Noise Models and Twirl Frame Errors

All experiments run via Qiskit 2.x (`AerSimulator`). In 4-qubit TFIM circuits, single-qubit gates (`rx, rz, h`) experience $p_1 = 10^{-3}$ depolarizing noise and coherent $R_Z(0.5\epsilon)$ phase drift, while CX gates incur $p_2 = 1.2 \times 10^{-2}$ depolarizing error plus coherent $R_{ZZ}(\epsilon) \cdot (I \otimes R_Z(\epsilon))$. To test whether Pauli Twirling gains rely on noiseless frame insertions, we additionally evaluate a **noisy-twirl-frame** configuration at $\epsilon = 0.08$ rad where inserted Pauli frame gates (`x, y, z`) carry depolarizing error $p_1 = 10^{-3}$ and coherent shift $R_Z(0.25\epsilon)$.

2.2 Fractional Folding, Clifford Twirling, and Budget Accounting

Non-uniform fractional unitary folding [4] scales CX vs. 1-qubit gates at a $10 : 1$ weight behind compiler barriers while preserving $U_{\text{folded}} = U$, yielding $\lambda \in \{1.000, 1.398, 1.796, 2.294, 2.851, 3.498\}$ on 4-qubit TFIM.

Every seed enforces a matched shot ceiling ($N_{\text{total}} = 24{,}000$ in 4Q TFIM; 32,768 in 8-scale 1Q; 24,576 in 6-scale 1Q):

- `Unmitigated_Equal_Budget`: 1 circuit ($\lambda = 1$) $\times N_{\text{total}}$ shots.
- `Pauli_Twirl_Only_Equal_Budget`: 12 twirled circuits ($\lambda = 1$) $\times 2{,}000 = 24{,}000$ shots.
- Untwirled ZNE (Quadratic / Exp / Harmonic / Auto): 6 scaled circuits $\times 4{,}000 = 24{,}000$ shots (Local Linear uses the first 4 scales, 16,000 shots).
- Pauli-Twirled ZNE (`Pauli_Twirl_Linear/Exp`): 4 scales $\times 12$ twirls $= 48$ circuits $\times 500 = 24{,}000$ shots.

2.3 Stability-Gated Lomb–Scargle Extrapolator

The 4-parameter model $E(\lambda) = E_0 e^{-\gamma\lambda}\cos(\omega\lambda + \phi)$ fits bounded `curve_fit` parameters $E_0 \in [-2, 2]$, $\gamma \in [0, 1.5]$, $\omega \in [0.01, 3.0]$, $\phi \in [-0.35, 0.35]$. Because Scargle's asymptotic exponential False-Alarm Probability has a hard small-N floor (FAP ≥ 0.137 at $N = 8$), `Auto_LombScargle_ZNE` triggers harmonic candidate fitting when FAP < 0.05 or ($N \leq 12$ and explained variance $R^2_{\text{peak}} = 2z_{\max}/N \geq 0.60$), and **accepts** the harmonic fit if and only if `unstable_fit` is `False`:

$$\texttt{unstable_fit} \equiv \texttt{at_bounds} \vee \left(\kappa(C_{\text{cov}}) > 10^4\right) \vee \neg\text{isfinite}(\kappa), \tag{1}$$

where $\kappa(C_{\text{cov}})$ is the covariance condition number and `at_bounds` flags upper bounds within 2% of span, non-zero lower bounds within 2%, or $\gamma \leq 10^{-4}$. Unstable harmonic candidates automatically fall back to `Local_Linear_ZNE`.

3 Benchmark Results

3.1 Regime A: In-Family Niche vs. Out-of-Family Breakdown

Table 1 demonstrates both the genuine single-mode niche and its fragility outside that family:

- **A1 (In-Family Deep Mono-Mode):** `Auto_LombScargle_ZNE` selects the harmonic model on 100% of seeds with 0% instability, lowering MAE from 0.0482 ± 0.0004 (`Unmitigated_Equal_Budget`) to 0.0084 ± 0.0012 (-82.6%, $z = +31.74$ SE).
- **A2–A4 (Short Scales, Monotonic Relaxation, and Bi-Chromatic $1/f$ Drift):** Forced Harmonic ZNE suffers 100% optimizer instability across all three regimes (MAE = 0.3601 in A2, 0.2956 in A3, 0.2007 in A4). The stability gate prevents catastrophic divergence (0% instability, 0% harmonic selection) by falling back to local linear extrapolation. While this matches monotonic ZNE performance in A3 (MAE = 0.0044 ± 0.0006 vs. Exponential 0.0037, $z = +40.47$ SE over equal-budget unmitigated), local linear fallback itself loses to `Unmitigated_Equal_Budget` when noise trajectories bend within the first 4 scales without reaching an asymptotic tail (A2: 0.0947 vs. 0.0487; A4: 0.1667 vs. 0.0904). No mitigation beats quadratic or unmitigated on turned/beating A2/A4 trajectories uniformly via simple linear fitting.

3.2 Regime B: 4-Qubit TFIM, Equal-Budget Baselines, and Noisy Twirl Frames

Table 2 and Figure 1 isolate what drives error reduction in multi-qubit entangled circuits:

1. **Harmonic ZNE Divergence and Stability Gate Recovery:** Forced Harmonic ZNE triggers 100% instability at all five 4Q TFIM settings (MAE = 1.2701 at $\epsilon = 0$, 2.2868 at $\epsilon = 0.04$), whereas `Auto_LombScargle_ZNE` maintains 0% instability by falling back cleanly to local linear ZNE (0.0194–0.1002).
2. **Twirling Only vs. Twirling + ZNE vs. Equal-Budget Unmitigated:** Merely executing Pauli Twirling at $\lambda = 1$ without extrapolation (`Pauli_Twirl_Only_Equal_Budget`, 12 circuits) already cuts $\epsilon = 0.08$ error from 0.2883 to 0.0991 (-65.6%). Adding ZNE across 4 scales (48 circuits) further lowers error to 0.0714 ± 0.0082 (-75.2% vs. equal-budget unmitigated, $z = +25.89$ SE), though at high $\epsilon = 0.14$ rad untwirled `Linear_ZNE` (0.1002) performs comparably to `Pauli_Twirl_Exp_ZNE` (0.0997, which shows 20% exponential fit instability).
3. **Robustness to Finite Twirl Frame Gate Errors:** Adding realistic depolarizing and coherent errors directly to inserted `x, y, z` twirl gates at $\epsilon = 0.08$ rad leaves `Pauli_Twirl_Linear_ZNE` at MAE = 0.0383 ± 0.0070 (-86.8% vs. equal-budget unmitigated 0.2911 ± 0.0012, $z = +35.55$ SE, 0% instability), confirming that Pauli Twirling remains advantageous even without ideal zero-error frame gates.

4 Discussion and Operational Conclusions

Our audit demonstrates that Harmonic ZNE does *not* provide a general-purpose NISQ advantage: its 82.6% MAE reduction (vs. equal-budget unmitigated) is strictly confined to well-sampled, single-mode coherent drift (A1). On short scales (A2), monotonic relaxation (A3), bi-chromatic $1/f$ drift (A4), and multi-qubit non-commuting TFIM circuits, forced 4-parameter harmonic fitting hits parameter bounds and diverges with 100% instability. Adding explicit covariance-conditioning and bound-saturation gates (`Auto_LombScargle_ZNE`) eliminates divergence safely, while Clifford Pauli Twirling combined with low-order ZNE provides substantial reductions (75.2%–86.8%) over equal-shot unmitigated and twirl-only baselines—at an explicit compilation cost of 48 circuit variants per seed versus 1 unmitigated or 12 twirl-only circuits.

Code and Data Availability

Code, CI tests (7/7 `pytest`), JSON audit tables, and manuscript sources are archived under Zenodo

Table 1: **Regime A (1-Qubit Echo, $K = 25$ Seeds, $E_{\text{ideal}} = +1.0000$): Out-of-Family Stress Tests and Equal-Budget Baselines.** Values show MAE ± SE; parenthetical percentages indicate optimizer instability rates (`inst`) or automatic harmonic selection rate (`sel`) for `Auto_LombScargle_ZNE`.

Method	**A1: Deep Mono-Mode** ($s \in \{1..17\}$, 32,768 shots)	**A2: Short Coherent** ($s \in \{1..6\}$, 24,576 shots)	**A3: Amp. Damp + Depol** ($\theta = 0$, $\gamma_{\text{AD}} = 0.015$)	**A4: Bi-Chromatic + $1/f$** ($\theta \in \{0.22, 0.53\}$, $\sigma_\theta = 0.07$)
Unmitigated (1 scale, 4,096 shots)	0.0491 ± 0.0011	0.0496 ± 0.0009	0.0318 ± 0.0007	0.0892 ± 0.0014
Unmitigated Equal Budget (1 circ.)	0.0482 ± 0.0004	0.0487 ± 0.0004	0.0324 ± 0.0002	0.0904 ± 0.0009
Global Quadratic ZNE	0.3199 ± 0.0011	$\mathbf{0.0349 \pm 0.0030}$	0.0046 ± 0.0006	$\mathbf{0.0387 \pm 0.0062}$
Local Linear ZNE (4 circ.)	0.1197 ± 0.0018	0.0947 ± 0.0018	0.0044 ± 0.0006	0.1667 ± 0.0041
Exponential ZNE	0.5766 ± 0.0014 (0%)	0.2758 ± 0.0026 (0%)	$\mathbf{0.0037 \pm 0.0004}$ (0%)	0.6979 ± 0.0063 (0%)
Damped Harmonic ZNE (Forced)	$\mathbf{0.0084 \pm 0.0012}$ (0% inst)	0.3601 ± 0.1899 (100% inst)	0.2956 ± 0.1612 (100% inst)	0.2007 ± 0.0129 (100% inst)
Auto Lomb–Scargle ZNE	$\mathbf{0.0084 \pm 0.0012}$ (100% sel)	0.0947 ± 0.0018 (0% sel)	0.0044 ± 0.0006 (0% sel)	0.1667 ± 0.0041 (0% sel)
Auto vs. Equal-Budget Unmit.	$\mathbf{-82.6}\%$ ($z = +31.74$ SE)	-94.5% worse ($z = -24.58$)	$\mathbf{-86.4}\%$ ($z = +40.47$ SE)	-84.4% worse ($z = -17.97$)

Table 2: **Regime B (4-Qubit TFIM Trotter, $\langle Z_0 \rangle_{\text{ideal}} = -0.4081$, $K = 20$ Seeds, $N_{\text{total}} = 24{,}000$ Shots/Seed).** Compares equal-budget unmitigated and twirl-only baselines under ideal Pauli frames ($\epsilon \in \{0.00..0.14\}$) and noisy Pauli frames (`x,y,z` with $p_1 = 0.001$, $R_Z(0.25\epsilon)$ at $\epsilon = 0.08$). Instability rates appear in brackets when non-zero.

Method *(Circuits / Shots)*	$\epsilon = 0.00$ rad	$\epsilon = 0.04$ rad	$\epsilon = 0.08$ rad	$\epsilon = 0.14$ rad	$\epsilon = 0.08$ **(Noisy Frames)**
Unmitigated (1 circ., 4k)	0.0273 ± 0.0027	0.1484 ± 0.0027	0.2872 ± 0.0032	0.5347 ± 0.0028	0.2848 ± 0.0026
Unmitigated Equal Budget (1 / 24k)	0.0265 ± 0.0014	0.1450 ± 0.0008	0.2883 ± 0.0017	0.5322 ± 0.0011	0.2911 ± 0.0012
Pauli Twirl Only Eq. Budget (12 / 24k)	0.0259 ± 0.0013	0.0565 ± 0.0037	0.0991 ± 0.0045	0.1554 ± 0.0084	0.1167 ± 0.0065
Local Linear ZNE (4 / 16k)	0.0194 ± 0.0036	0.0407 ± 0.0051	0.0735 ± 0.0056	0.1002 ± 0.0056	0.0817 ± 0.0063
Quadratic Richardson ZNE (6 / 24k)	0.0396 ± 0.0076	0.1027 ± 0.0089	0.2086 ± 0.0101	0.2006 ± 0.0086	0.2370 ± 0.0086
Exponential ZNE (6 / 24k)	$\mathbf{0.0156 \pm 0.0027}$	0.3763 ± 0.0157	0.5405 ± 0.0017 [1.0]	0.7876 ± 0.0012 [1.0]	0.5413 ± 0.0013 [1.0]
Pauli Twirl + Linear ZNE (48 / 24k)	0.0243 ± 0.0035	$\mathbf{0.0347 \pm 0.0042}$	0.0766 ± 0.0105	0.1212 ± 0.0158	$\mathbf{0.0383 \pm 0.0070}$
Pauli Twirl + Exp ZNE (48 / 24k)	0.0307 ± 0.0047	0.0363 ± 0.0044	$\mathbf{0.0714 \pm 0.0082}$ [0.25]	$\mathbf{0.0997 \pm 0.0136}$ [0.20]	0.0406 ± 0.0069
Damped Harmonic ZNE (6 / 24k)	1.2701 ± 0.1530 [1.0]	2.2868 ± 0.0000 [1.0]	0.3042 ± 0.0650 [1.0]	0.5550 ± 0.0018 [1.0]	0.2493 ± 0.0628 [1.0]
Auto Lomb–Scargle ZNE (6 / 24k)	0.0194 ± 0.0036	0.0407 ± 0.0051	0.0735 ± 0.0056	0.1002 ± 0.0056	0.0817 ± 0.0063
Harmonic 2-SE Niche Confirmed?	False ($z = -8.20$)	False ($z = -531.4$)	False ($z = -3.55$)	False ($z = -33.25$)	False ($z = -3.34$)
Best Gain vs. Eq. Budget Unmit.	-41.1% (Exp)	$\mathbf{-76.1}\%$ (Tw+Lin)	$\mathbf{-75.2}\%$ (Tw+Exp)	$\mathbf{-81.3}\%$ (Tw+Exp)	$\mathbf{-86.8}\%$ (Tw+Lin)

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References

[1] J. Preskill, Quantum **2**, 79 (2018).
[2] K. Temme, S. Bravyi, and J. M. Gambetta, Phys. Rev. Lett. **119**, 180509 (2017).
[3] Y. Li and S. C. Benjamin, Phys. Rev. X **7**, 021050 (2017).
[4] T. Giurgica-Tiron, Y. Hindy, R. LaRose, A. Mari, and W. J. Zeng, in *2020 IEEE QCE* (IEEE, 2020), pp. 306–316.
[5] A. Kandala et al., Nature **567**, 491 (2019).
[6] J. J. Wallman and J. Emerson, Phys. Rev. A **94**, 052325 (2016).
[7] A. Hashim et al., Phys. Rev. X **11**, 041039 (2021).
[8] N. R. Lomb, Astrophys. Space Sci. **39**, 447 (1976).
[9] J. D. Scargle, Astrophys. J. **263**, 835 (1982).

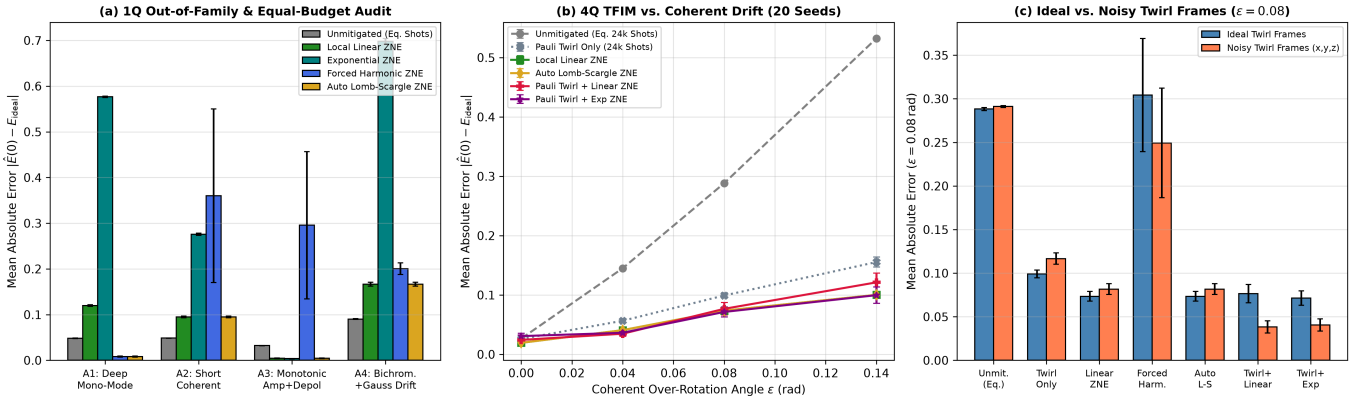

Figure 1: **Equal-budget, out-of-family, and noisy-frame QEM audit.** (a) 1-qubit Regime A1–A4 stress test showing Forced Harmonic ZNE breaking down (100% instability) in A2–A4 while `Auto_LombScargle_ZNE` isolates A1 (0.0084) and safely avoids divergence. (b) 4-qubit TFIM MAE vs. coherent angle ϵ comparing `Unmitigated_Equal_Budget`, `Pauli_Twirl_Only_Equal_Budget`, `Auto_LombScargle_ZNE`, and Pauli-Twirled ZNE. (c) Ideal vs. noisy Pauli twirl frame (`x, y, z`) comparison at $\epsilon = 0.08$ rad.